Spatial variances of wind fields and their relation to second-order structure functions and spectra

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Numerical Weather Prediction

Introduction

- Working with Greg King on 2nd and 3rd order structure functions (see posters on Atmosphere and Ocean Coupling)
- Common knowledge: $\sigma^2(r) = \frac{1}{2}D(r)$
- Doubts, because this is only valid if the autocorrelation goes to zero (very large distances), which is not the case for ocean surface vector winds
- Searched and found a better measure: spatial variances
- Relation between spatial variances and 2nd order structure functions (*Yates*, 1948)
- But spectrum can not be interpreted as a variance density

Spectra

For a continuous function $u(r)$ the spectrum is defined as

$$\psi(k) = \left| \int_{-\infty}^{\infty} dr\, e^{2\pi ikr} u(r) \right|^2$$

or as the Fourier transform of its autocovariance

$$\psi(k) = \int_{-\infty}^{\infty} dr\, e^{2\pi ikr} A(r)$$

$$A(r) = \int_{-\infty}^{\infty} ds\, u(s) u(s+r) \qquad A(0) = \int_{-\infty}^{\infty} ds\, u^2(s) = \sigma^2$$

with normalisation

$$\sigma^2 = \int_{-\infty}^{\infty} dk\, \psi(k)$$

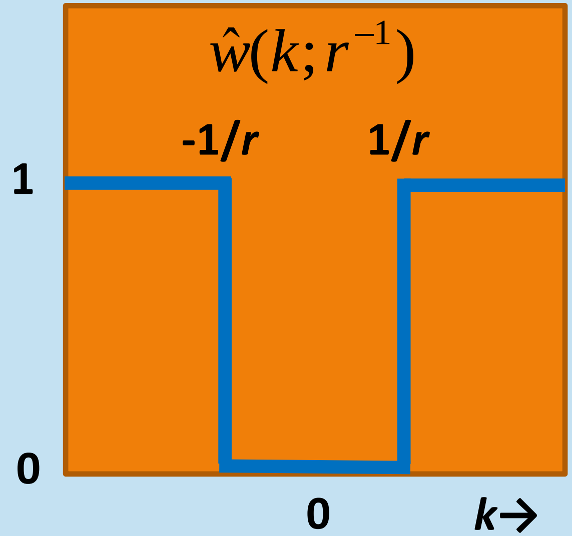
This is the only justification for the interpretation as variance density

Spectra and variance

Suppose the spectrum is a variance density, then

$$\sigma^2(r) = \int_{r^{-1}}^{\infty} dk\, \Psi(k) = \int_{r^{-1}}^{\infty} dk \left[\psi(k) + \psi(\text{-}k)\right] = \int_{-\infty}^{\infty} dk\, \psi(k)\hat{w}(k;r^{-1})$$

with $\hat{w}$ a high-pass filter $\hat{w}(k;r^{-1}) = \begin{cases} 1 & |k| \geq r^{-1} \\ 0 & |k| < r^{-1} \end{cases}$



Fourier transform:

$$\sigma^2(r) = \int_{-\infty}^{\infty} dy\, A(y) w(y;r^{-1})$$

$$w(y;r^{-1}) = \frac{\sin\left(\frac{2\pi}{r} y\right)}{\pi y}$$

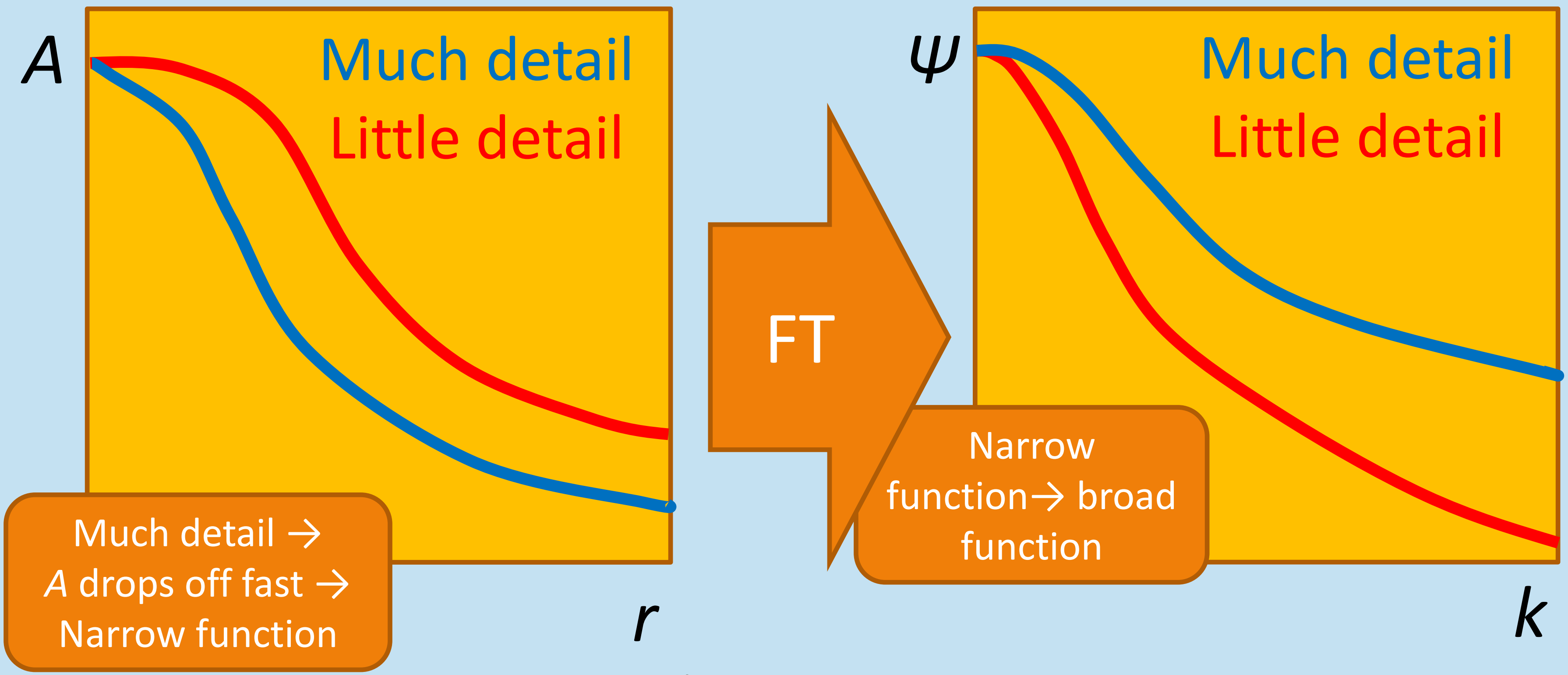
Not a variance in position space, but the integral of the autocovariance weighted with a sinc-function.

When $r \to \infty \quad \Leftrightarrow \quad r^{-1} \to 0 \quad \Rightarrow \quad w(y;r^{-1}) \to \delta(y)$

Spectra → variance

Interpretation of spectrum as variance density qualitatively correct

Two datasets, one with much small-scale detail (e.g., scat), and one with little small-scale detail (e.g. ECMWF)



Structure functions

For a discrete dataset $\{u_i\} = \{u(r_i)\},\ r_i = i\Delta r$

the 2nd order structure function is defined as

$$D(r_n) = \left\langle \left(u_i - u_{i+n}\right)^2 \right\rangle$$

with the averaging <.> over all samples

In case of scatterometer winds u stands for the wind component parallel or perpendicular the sampling direction, which in general is along track

Advantages:

- tolerant for missing points
- easy to implement

Structure functions and variance

It is easily shown that

$$D(r) = 2\sigma^2 \left[1 - \rho(r)\right]$$

with $\rho(r)$ the autocorrelation. For large distances:

$$r \to \infty \quad \Rightarrow \quad \rho(r) \to 0 \quad \Rightarrow \quad D \to 2\sigma^2$$

Not in real life for scatterometer winds!

Moreover, for neighboring points $D(r_1) = 4\sigma_1^2$

Spatial variance

The variance of a sample $\{u_i\}; i = 0,1,\cdots,n$

$$\sigma_n^2 = \frac{1}{n+1}\sum_{i=0}^{n}(u_i - \bar{u})^2$$

definition

$$\sigma_n^2 = M_2 - M_1^2 \quad , \quad M_m = \frac{1}{n+1}\sum_{i=0}^{n} u_i^m \quad , \quad M_1 = \bar{u}$$

calculation

$$\sigma_n^2 = \frac{1}{(n+1)^2}\sum_{\substack{i,j=0\\ j>i}}^{n}(u_i - u_j)^2$$

Yates eq.

Spatial variance: $V(r_n) = \langle \sigma_n^2 \rangle$

Yates (1948): $V(r_n) = \frac{1}{(n+1)^2}\sum_{m=1}^{n}(n+1-m)D(r_m)$

Spatial variance and sampling

Spatial variances:

- have clear interpretation
- are tolerant for missing points
- are easily implemented

but

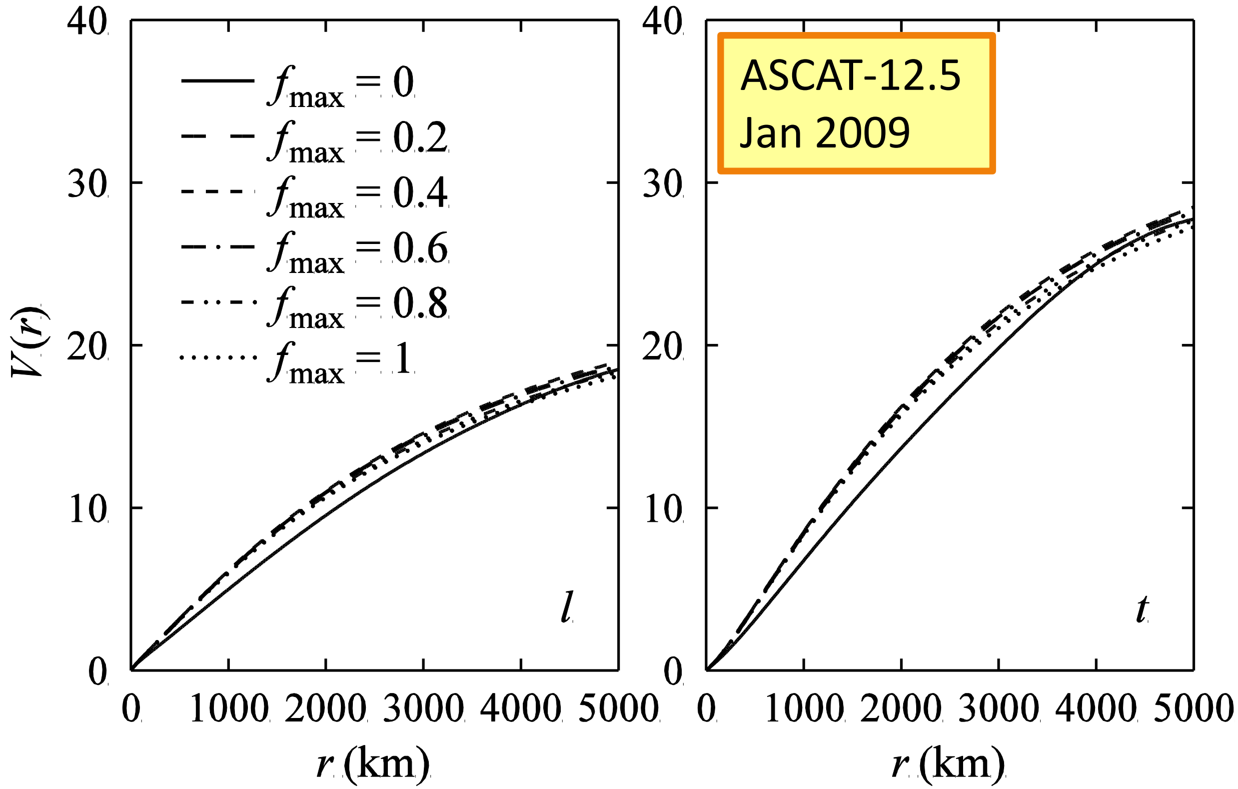
- depend on sampling strategy

Sampling strategies:

- non-overlapping samples without missing points (spectra)
- overlapping samples, all points accepted (structure functions)
- maximum fraction of missing points
- weight dependent on number of missing points in sample

Sampling strategy example

Spatial variance against *r*



Sampling strategy has small effect

Overlapping samples

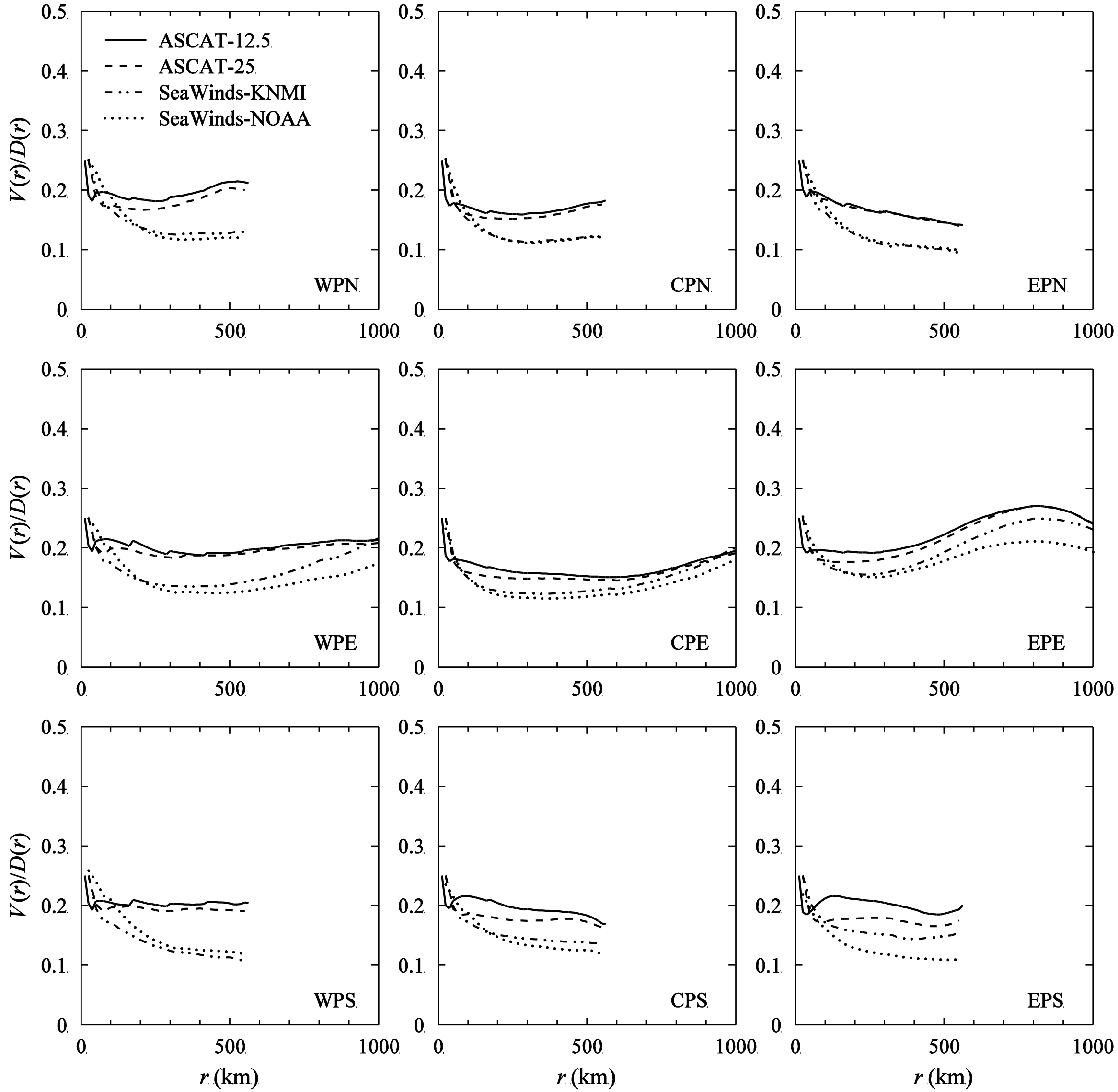
Maximum fraction of missing points f_{max}

Relative weight

$$w = \frac{n - m}{n}$$

m: number of missing points,
n: sample size

Structure functions as proxy for variance



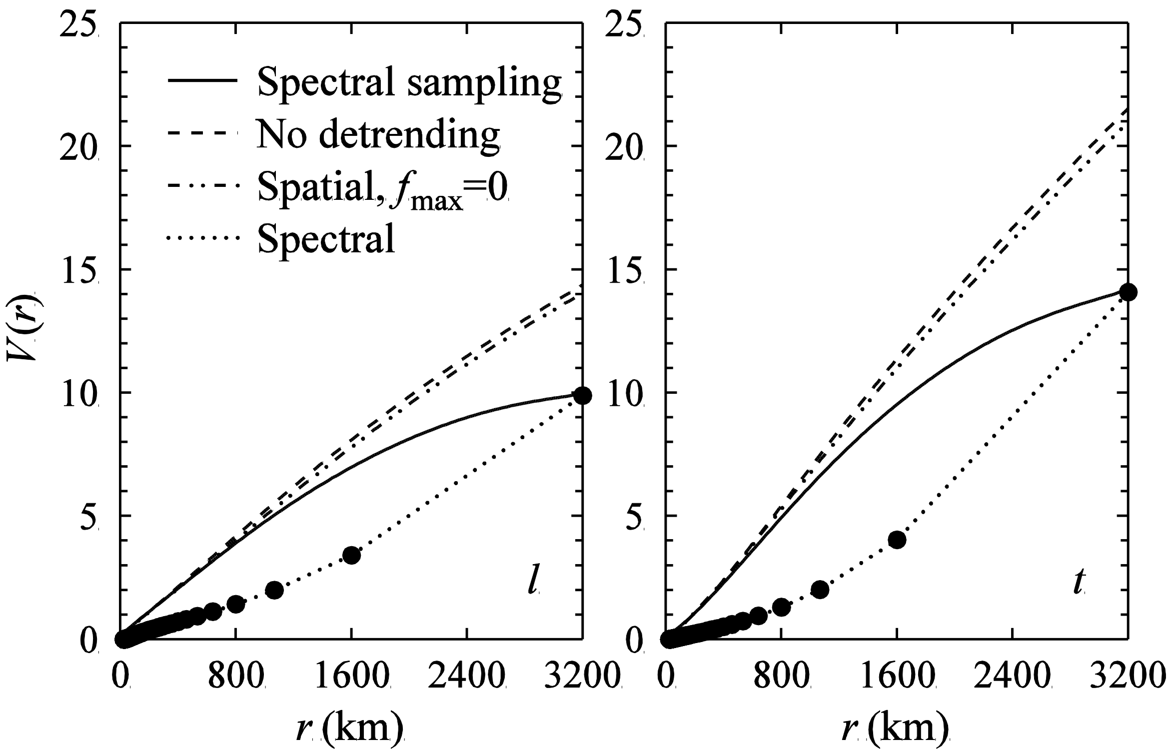
Tropical Pacific
(see posters G. King)

ASCAT-12.5
ASCAT-25
SeaWinds-KNMI
SeaWinds-NOAA

January 2009

$$\sigma^2(r) \approx \tfrac{1}{5} D(r)$$

Comparison with spectral variance



Spectral sampling:
non-overlapping and detrended samples without missing points

Tropical Pacific:
non-overlapping samples without missing points, but not detrended

Spatial:
overlapping samples without missing points

Spectral:
assuming the spectrum as variance density with $r = k^{-1}$

- Spectral sampling not seriously biased to calm conditions
- Detrending OK for small scales
- Spectrum no variance density

Conclusions

- Neither spectra nor 2nd order structure functions represent variance as a function of scale well; spatial variance does.
- 2nd order structure functions good proxy for variance
- Spectral sampling not seriously biased to calm conditions
- Sampling strategy has relatively small effect (< 30%)

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To be submitted to JGR

Recommendations

- Use spatial variances to calculate representativeness errors (*first results in triple collocation: integration range 200 km instead of 800 km*)

- If you want variance as a function of scale: calculate variances!